

Math 10460 - Honors Mathematics II

Homework 9a - Due Wednesday, March 23

You must show your work in all of the problems!!!

- (1) Show that, for a line l which makes an angle ψ with the positive x -axis, the matrix for reflection about l is given by

$$S_l = S_\psi = \begin{pmatrix} \cos 2\psi & \sin 2\psi \\ \sin 2\psi & -\cos 2\psi \end{pmatrix}$$

Hint: Come up with unit vectors which are parallel and perpendicular to l , then do the construction from class. A *unit vector* is a vector $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ such that its length

$$\|\vec{v}\| = \sqrt{a^2 + b^2} = 1.$$

- (2) Show that $S_\psi^{-1} = S_\psi$.
- (3) Show that $R_\theta S_\psi = S_\psi R_\theta^{-1}$ for any θ and ψ .
- (4) We know that $R_\theta S_\psi = S_\varphi$ for some angle φ (the composition of a rotation and a reflection is just a reflection about some other line). Find φ in terms of θ and ψ .
- (5) Consider the square bounded by the lines $x + y = 2$, $x + y = -2$, $x - y = 2$, and $x - y = -2$. Four of the symmetries of this square are given by reflections. Write down matrices for all four of these reflections. *Hint:* Recall that reflections take the form $r^a s$ for some rotation r and an appropriate reflection s .